New Methodology for ATMS Noise Characterization

Miao Tian¹, Cheng Da², Xiaolei Zou¹, and Fuzhong Weng³

1. ESSIC, University of Maryland, College Park
2. Dept. of EOAS, Florida State University
3. NOAA/NESDIS/STAR

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Currently, the Noise Equivalent Differential Temperature\(^1\) (NEDT) calculated based on the method of **standard deviation** is commonly used for the noise characterization of operational satellite instruments.

- **Issues within the current operational NEDT are:**
  1. Determination of optimal sample size (i.e., averaging window size)
  2. Overestimation has been noted
  3. The standard deviation is valid for a data with stable mean

- **This research proposes the **Allan deviation\(^1,2\)** as a candidate for NEDT**
  1. The form of Allan deviation is determined from
     a) The overlapping Allan deviation\(^1\)
     b) The two-sample (i.e., neighborhood) Allan deviation
  2. Application to the ATMS on-orbit calibration warm counts
     a) Allan deviation vs. the sample size
     b) NEDT calculated by the Allan deviation and standard deviation

The current operational NEDT is calculated via

$$\text{NEDT}_{ch} = \sqrt{\frac{1}{4M} \sum_{i=1}^{M} \sum_{j=1}^{4} \left( \frac{C^w_{ch}(i, j) - \overline{C^w_{ch}(i)}}{G_{ch}} \right)^2}$$

(1)

where $C^w_{ch}$ represents the warm counts per channel and $G_{ch}$ is the averaged calibration gain. In ATMS, the warm counts are measured at 4 scan positions.

By using the **overlapping Allan deviation**\(^1\), NEDT is calculated via

$$\sigma_{ch}^{\text{Allan}} = \sqrt{\frac{1}{2m^2(M - 2m + 1)} \sum_{i=1}^{M-2m+1} \sum_{k=i}^{i+m-1} \left[ \frac{C^w_{ch}(k + m) - C^w_{ch}(k)}{G_{ch}} \right]^2}$$

(2)

When $m = 1$, (2) becomes the NEDT by the **two-sample Allan deviation**

$$\sigma_{ch}^{\text{Allan}} = \sqrt{\frac{1}{2(M - 1)} \sum_{i=1}^{M-1} \left[ \frac{C^w_{ch}(i + 1) - C^w_{ch}(i)}{G_{ch}} \right]^2}$$

(3)
Theoretical Study on Allan Deviation
The two Allan deviations (eqs.2-3) are tested on the following two datasets:

Data 1: Stationary data: constant + Gaussian noise, $N(0,1)$
Data 2: Non-stationary data: sine + Gaussian noise, $N(0,1)$

**Conclusion:** The overlapping Allan deviation cannot accurately estimate the noises in both cases while the two-sample Allan deviation can provide estimates that are very close to the noise standard deviation in both cases.
Studies on Allan Deviation (2)

- Assuming independent time series with a distribution of $N(\mu, \sigma)$, by taking expectation on both sides of (3), it yields:

$$E(\sigma_{Allan}^2) = \frac{2(M - 1)(\sigma^2 + \mu^2) - 2(M - 1)\mu^2}{2(M - 1)}$$

For the cases like Data 1, $E(\sigma_{Allan}^2) = \sigma^2$, which proves that the two-sample Allan variance gives an unbiased estimate of the noise true variance.

- The non-stationary case is further explored in a numerical way:

  Sinusoidal signal: $y(x) = \alpha \sin(\omega x), x = 1,2, \ldots, 1000$

  Gaussian noise: $\varepsilon \sim N(0,1)$

  - $\alpha$ varies from $10^{-3}$ to $10^3$ and $\omega$ varies from $10^0$ to $10^4$
  - $\sigma_{Allan}$ and $\sigma_{std}$ are calculated for each pair of $(\alpha, \omega)$ and compared in the following two forms:

    a) $\frac{\sigma_{Allan} - \sigma_{noise}}{\sigma_{noise}}$ and b) $\frac{\sigma_{std} - \sigma_{noise}}{\sigma_{noise}}$
When $\alpha$ is less than 10, both methods can provide estimates that are very close to the true noise standard deviation within 10% of error.

As $\alpha$ increases, the relative errors increase in both methods. However, the area with errors less than 10% from the two-sample Allan deviation is much larger than that from the standard deviation.
Application to the ATMS On-Orbit Data
ATMS On-orbit Warm Counts

Ch. 14

Ch. 15

Ch. 16

Ch. 22
Two-Sample Allan Deviation is applied here. Warm Counts from different scan positions are given in different color lines and their mean is given in the black line.
Two-Sample Allan Deviation is applied here. Warm Counts from different scan positions are given in different color lines and their mean is given in the black line.
NEDT by Two-Sample Allan and Std. Dev.

Blue: Std. dev.  Red: Allan dev.
Stability Study

Warm counts used in calculation:
100 - pink,
300 - blue,
1000 - orange,
2034 - green
Summary and Future Work

1. The two-sample Allan deviation is introduced as an alternative approach for characterizing the ATMS channel precision, NEDT.

2. From our analysis, firstly it shows that the overlapping Allan deviation cannot accurately estimate the noise magnitudes of both stationary and non-stationary cases.

3. The research proves the two-sample Allan variance will provide an unbiased estimate of a stationary data. Also it shows the two-sample Allan deviation always give an estimate closer to the true noise standard deviation comparing with the standard deviation.

4. From the ATMS data, it shows that the NEDTs of the water vapor channels, 16-22, have large differences between the Allan deviation and standard deviation. Moreover, it also shows that overall the NEDT calculated by the Allan deviation is more stable w.r.t the sample size, than that by the standard deviation.

Hence, the two-sample Allan deviation is recommended for noise characterization of ATMS and other satellite microwave sensors.
Questions?